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Flow induced by the presence of a non-conducting ellipsoid of revolution in fluid carrying a uniform current

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The Stokes creeping flow, induced by the passage of a uniform current parallel to the axis of a stationary non-conducting ellipsoid of revolution in an incompressible viscous fluid occupying, apart from the ellipsoidal region, the whole space, is investigated. The magnetic field, which is due to the distortion of the uniform current by the ellipsoid, is zero all over the surface of the ellipsoid. The induced flow field is symmetric with respect to the axis, and also with respect to a plane through the centre perpendicular to the axis of the ellipsoid. The case of a non-conducting circular disk, with its plane perpendicular to the direction of the undisturbed current, is deduced from that of a planetary ellipsoid.

1. Introduction

When a conducting fluid is permeated by a uniform current, which is distorted by the fact that different fluid regions have different conductivities, a magnetic field is set up. The resultant Lorentz force is, in general, rotational and cannot be balanced by a hydrostatic pressure and thus the fluid is set in motion. If the conducting fluid possesses a general motion, then the Lorentz force, due to the passage of a current and the associated magnetic field, will modify the original flow field.

The steady-state flow field, set up when a uniform current is passed through a conducting infinite fluid, in the presence of a non-conducting sphere, was investigated by Chow (1966). Here we extend this investigation. First, we consider the equations governing the steady state flow set up when the uniform current in an infinite fluid is distorted by the presence of an axially symmetric body having its axis along the direction of the undisturbed current. We then apply our analysis to the cases when the symmetric body is an ovary and a planetary ellipsoid.

Magnetohydrodynamic problems are in general very difficult and simplifying approximations are used in order to obtain mathematical solutions to these problems. Thus, the effect of a strong flow field on a relatively weak electromagnetic field may be found by assuming that the effect of the latter on the former is negligible. Similarly, an approximation to the effect of a strong electromagnetic field on a relatively weak flow field may be obtained, by assuming that the former is unaffected by the latter. Here we assume that the flow field is weak and use the second approximation. We also neglect the inertia terms from the momentum equation, that is, we obtain a Stokes solution, which is not very accurate far away from the body.

2. General equations of the problem

We consider an infinite incompressible viscous conducting fluid, carrying a uniform current $\mathbf{J}_0$ which is disturbed by the presence of an axially symmetric body that has its axis parallel to the direction of the current at infinity. The induced velocity is small and if we assume that its effect on the electromagnetic field is negligible, that is, if we assume that the currents are driven by the electric field, in the steady state, the electric current $\mathbf{J}$ and magnetic field $\mathbf{B}$ are connected by the equations

$$\nabla \times \mathbf{B} = 4\pi \mathbf{J}, \tag{1}$$

$$\nabla \times \mathbf{J} = 0. \tag{2}$$

The boundary conditions are that at the surface of the body the normal component of the current is zero and at infinity $\mathbf{J}$ tends to $\mathbf{J}_0$. Thus, the electric current satisfies the same equations and boundary conditions as the velocity field of an inviscid incompressible fluid past a fixed solid body.

We use cylindrical polar co-ordinates (r, θ, x) with the x axis along the axis of symmetry of the body and, as with inviscid flow, make use of a current stream function ψ_1, such that

$$\mathbf{J} = \frac{1}{4\pi r}\left(-\frac{\partial}{\partial x}, 0, \frac{\partial}{\partial r}\right)\psi_1. \tag{3}$$

From the symmetry of the problem it follows that the magnetic field lines are circles about the axis of symmetry; by using (1) and (3) we find

$$\mathbf{B} = \hat{\boldsymbol{\theta}}\psi_1(r, x)/r, \tag{4}$$

and hence

$$\mathbf{J} \times \mathbf{B} = -\frac{\psi_1}{4\pi r^2}\nabla\psi_1. \tag{5}$$

If we assume that the induced flow is slow enough, so that the inertia forces are negligible, the momentum equation in the steady state is

$$\nabla p + \mu_0 \nabla \times \nabla \times \mathbf{V} - \mathbf{J} \times \mathbf{B} = 0, \tag{6}$$

where p is the fluid pressure, $\mathbf{V}$ the fluid velocity and μ_0 the coefficient of viscosity.

If we express the velocity in terms of the stream function ψ_2 by

$$\mathbf{V} = (-\partial/r\,\partial x, 0, \partial/r\,\partial r)\,\psi_2 \tag{7}$$

and take the curl of (6) we find that ψ_1 and ψ_2 are connected by the equation

$$\psi_1\,\partial\psi_1/r^2\,\partial x = -2\pi\mu_0 D^4\psi_2, \tag{8}$$

where

$$D^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r}.$$

For any particular axisymmetric configuration we must first find ψ_1, that is, solve the equation

$$D^2\psi_1 = 0$$

and then solve (8) for ψ_2.

The velocity field may also be obtained as follows: Take the divergence of (6) and solve the resulting Poisson equation for p. Since $\nabla\times\nabla\times\mathbf{V} = -\nabla^2\mathbf{V}$, (6) shows that each velocity component (use Cartesian co-ordinates for this purpose) can be obtained by solving Poisson's equation.

If we do a similar analysis for two-dimensional configurations, we find that the force $\mathbf{J}\times\mathbf{B}$ is irrotational, that is, it simply readjusts the fluid pressure and does not induce any flow.

3. Flow around an ovary ellipsoid

Let a and c be the polar and equatorial radii, e the eccentricity of a meridional cross-section, and the x axis the axis of the ellipsoid. We make use of the transformation

$$x = \kappa\cos\theta\cosh\eta = \kappa\mu\zeta; \quad r = \kappa\sin\theta\sinh\eta = \kappa(1-\mu^2)^{\frac{1}{2}}(\zeta^2-1)^{\frac{1}{2}}, \tag{9}$$

where $\mu = \cos\theta$, $\zeta = \cosh\eta$ and κ is a constant. Thus our ellipsoid is given by $\eta = \eta_0$ or $\zeta = \zeta_0$, where

$$\kappa = ae, \quad \zeta_0 = 1/e, \quad \kappa(\zeta_0^2-1)^{\frac{1}{2}} = c. \tag{10}$$

At infinity let the impressed current be parallel to the x axis and have intensity J_0. The current stream function, obtained from formulas given by Lamb (1932), is

$$\psi_1 = 2\pi J_0\kappa^2\left[1-\tfrac{1}{2}B_0\left\{\log\left(\frac{\zeta+1}{\zeta-1}\right)-\frac{2\zeta}{\zeta^2-1}\right\}\right](1-\mu^2)(\zeta^2-1), \tag{11}$$

where $$B_0 = 2\bigg/\left[\log\left(\frac{1+e}{1-e}\right)-\frac{2e}{1-e^2}\right].$$

On making use of (9) and substituting the value of ψ_1 given by (11) into (8) we obtain

$$\left[\frac{1}{\zeta^2-\mu^2}\left\{(\zeta^2-1)\frac{\partial^2}{\partial\zeta^2}+(1-\mu^2)\frac{\partial^2}{\partial\mu^2}\right\}\right]^2\psi_2$$
$$= \frac{4\pi J_0^2 B_0\kappa^5}{\mu_0}\left[1-\tfrac{1}{2}B_0\left\{\log\left(\frac{\zeta+1}{\zeta-1}\right)-\frac{2\zeta}{\zeta^2-1}\right\}\right]\frac{\mu(1-\mu^2)}{\zeta^2-\mu^2}. \tag{12}$$

Now we note the following property of the Legendre polynomial, $P_n(\mu)$, of degree n

$$(1-\mu^2)[(1-\mu^2)P_n']'' = -n(n+1)(1-\mu^2)P_n'(\mu) \tag{13}$$

and therefore we construct a solution by setting

$$\left.\begin{aligned} &\psi_2 = h_0(\zeta)(1-\mu^2)P_2'(\mu)+h_1(\zeta)(1-\mu^2)P_4'(\mu), \\ \text{or}\quad &\psi_2 = [f_0(\zeta)+A-(7\mu^2-3)f_1(\zeta)]\mu(1-\mu^2), \end{aligned}\right\} \tag{14}$$

where A is an arbitrary constant. $\mu(1-\mu^2)$ is one of the solutions of the complementary function of (12); it is found convenient to separate A from f_0 here. In the above and subsequent expressions, dashes denote differentiation with respect to the obvious variable μ or ζ as the case may be.

If we now substitute (14) into (12), after some manipulation we obtain the following set of equations:

$$(\zeta^2-1)f_0''-6f_0 = (7\zeta^2-3)\,F(\zeta), \tag{15}$$

$$(\zeta^2-1)f_1''-20f_1 = F(\zeta), \tag{16}$$

where
$$(\zeta^2-1)\,F''-6F = A_0\left[1-\tfrac{1}{2}B_0\left\{\log\left(\frac{\zeta+1}{\zeta-1}\right)-\frac{2\zeta}{\zeta^2-1}\right\}\right] \tag{17}$$

and
$$A_0 = 4\pi J_0^2 B_0 \kappa^5/7\mu_0.$$

Thus we must first solve (17) and then (15) and (16). The boundary conditions are that the velocity vanishes on the ellipsoid, that is,

$$A+f_0(\zeta_0) = f_0'(\zeta_0) = f_1(\zeta_0) = f_1'(\zeta_0) = 0 \tag{18}$$

and the velocity is finite at infinity. This latter boundary condition is somewhat unrealistic and is derived from the principle of minimum singularity (Van Dyke 1964). The reason for the non-vanishing of the velocity field at infinity is due to the use of Stokes' approximation, and the neglect of the inertia terms from the momentum equation. If we take the curl of the momentum equation, retaining the inertia terms and using the $\mathbf{J}\times\mathbf{B}$ obtained here, and do an order of magnitude analysis we find that at a large distance L from the origin

$$\rho\,\frac{V^2}{L^2} \sim \frac{4\pi J_0^2 B_0 \kappa^5}{L^3}.$$

Therefore for large L
$$V = O\left[\left|\frac{4\pi J_0 B_0 \kappa^5}{L\rho}\right|^{\frac{1}{2}}\right]$$

and tends to zero as L tends to infinity. For the case of a uniform current carrying fluid streaming past a sphere, Chow & Billings (1967) retained inertia terms and carried out an analysis using the Stokes and Oseen expansions near and far from the body, respectively. They matched their expansions, using the methods developed by Kaplun & Lagerstrom (1957) and Proudman & Pearson (1957), and found that at a large distance L from the centre of the sphere the effect of the electromagnetic field on the induced velocity is $O(1/L)$.

Since the Stokes approximation is inaccurate at large distances from the body, our solution is valid only for distances not too far from the ellipsoid.

If we now note that $\zeta(\zeta^2-1)$ and $\zeta(\zeta^2-1)(7\zeta^2-3)$ are solutions of the complementary functions of (15) and (16), respectively, we are able to obtain the solutions of (17), (15) and (16) by direct integration.

The solution of (17) is

$$F = -A_0\left[\tfrac{1}{6}+C\left\{6\zeta^2-4-3\zeta(\zeta^2-1)\log\left(\frac{\zeta+1}{\zeta-1}\right)\right\}\right.$$
$$\left.+\frac{B_0}{8}\left\{4\zeta-2(2\zeta^2-1)\log\left(\frac{\zeta+1}{\zeta-1}\right)+\zeta(\zeta^2-1)\log^2\left(\frac{\zeta+1}{\zeta-1}\right)\right\}\right], \tag{19}$$

where C is an arbitrary constant, and the other constant of integration has been set equal to zero.

After lengthy integrations by parts we find that the solutions of (15) and (16) are

$$f_0 = A_0\left[\frac{21\zeta^2-13}{72} + C_1\left\{6\zeta^2 - 4 - 3\zeta(\zeta^2-1)\log\left(\frac{\zeta+1}{\zeta-1}\right)\right\}\right.$$
$$- C\zeta\left\{\zeta(3\zeta^2-4) - \tfrac{1}{2}(\zeta^2-1)(3\zeta^2-2)\log\left(\frac{\zeta+1}{\zeta-1}\right)\right\} - \tfrac{1}{4}B_0\zeta(\zeta^2-1)$$
$$\left. - \frac{B_0}{16}\left\{\zeta(\zeta^4-1)\log^2\left(\frac{\zeta+1}{\zeta-1}\right) - (\zeta^2-1)(4\zeta^2+3)\log\left(\frac{\zeta+1}{\zeta-1}\right) + 2\zeta\right\}\right] \tag{20}$$

and

$$f_1 = A_0\left[\tfrac{1}{120} + C_2\left\{210\zeta^4 - 230\zeta^2 + 32 - 15\zeta(\zeta^2-1)(7\zeta^2-3)\log\left(\frac{\zeta+1}{\zeta-1}\right)\right\}\right.$$
$$+ \frac{C}{16}\left\{3\zeta(\zeta^2-1)(1-5\zeta^2)\log\left(\frac{\zeta+1}{\zeta-1}\right) + 2\zeta^2(15\zeta^2-13)\right\}$$
$$- \frac{B_0}{128}\left\{\zeta(\zeta^2-1)(9\zeta^2-5)\log^2\left(\frac{\zeta+1}{\zeta-1}\right)\right.$$
$$\left.\left. - 4(\zeta^2-1)(9\zeta^2-2)\log\left(\frac{\zeta+1}{\zeta-1}\right) + 4\zeta(9\zeta^2-8)\right\}\right]. \tag{21}$$

The constants C, C_1 and C_2 in the above equations and A in (14) are obtained from the boundary conditions (18). In (20) $-\frac{1}{4}B_0\zeta(\zeta^2-1)$ belongs to the complementary function; the constant $-\frac{1}{4}B_0$ was suitably chosen so that the velocity at infinity is finite.

As e approaches 1, ζ_0 tends to 1 and the ellipsoid tends to become an elongated rod. In this case B_0 tends to zero and thus the electromagnetic force and its flow effects become negligible, as expected.

As noted by Chow (1966) for a sphere, the force $\mathbf{J}\times\mathbf{B}$ and the induced creeping flow are symmetrical, not only with respect to the x axis, but also with respect to the plane $\theta = \frac{1}{2}\pi$, and therefore the induced drag effects on the ellipsoid are zero.

Stokes streaming flow of a viscous fluid with constant low velocity U at infinity, parallel to the axis of a fixed ovary ellipsoid, is discussed by Happel & Brenner (1965). The stream function ψ_0 for such a flow, streaming parallel to the positive x axis at infinity, is given by

$$\psi_0 = \tfrac{1}{2}U\kappa^2(\zeta^2-1)(1-\mu^2)\left[1 - \frac{(\zeta_0^2+1)\log\left(\dfrac{\zeta+1}{\zeta-1}\right)\Big/(\zeta_0^2-1) - 2\zeta/(\zeta^2-1)}{(\zeta_0^2+1)\log\left(\dfrac{\zeta_0+1}{\zeta_0-1}\right)\Big/(\zeta_0^2-1) - 2\zeta_0/(\zeta_0^2-1)}\right]. \tag{22}$$

If the fluid is conducting and we impose a uniform current J_0 parallel to the x axis at infinity, since U is small we may again neglect the effect of the flow field on the electromagnetic field. The total flow field is then due to the superposition of the uniform streaming flow and the electromagnetically induced flow and therefore its stream function ψ is given by

$$\psi = \psi_0 + \psi_2. \tag{23}$$

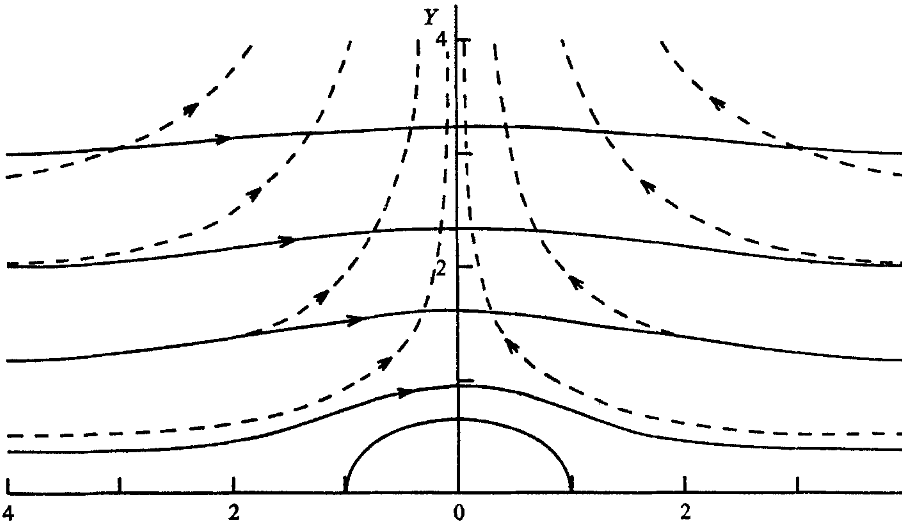

FIGURE 1. Streamlines (flow lines) in the upper half plane of a meridian section of an ovary ellipsoid for various values of ψ. ———, $\psi = \psi_0$; - - -, $\psi = \psi_2$. $e = 0{\cdot}75$. $X = x/a$, $Y = y/a$. The uniform stream or current is directed from the negative to the positive X axis.

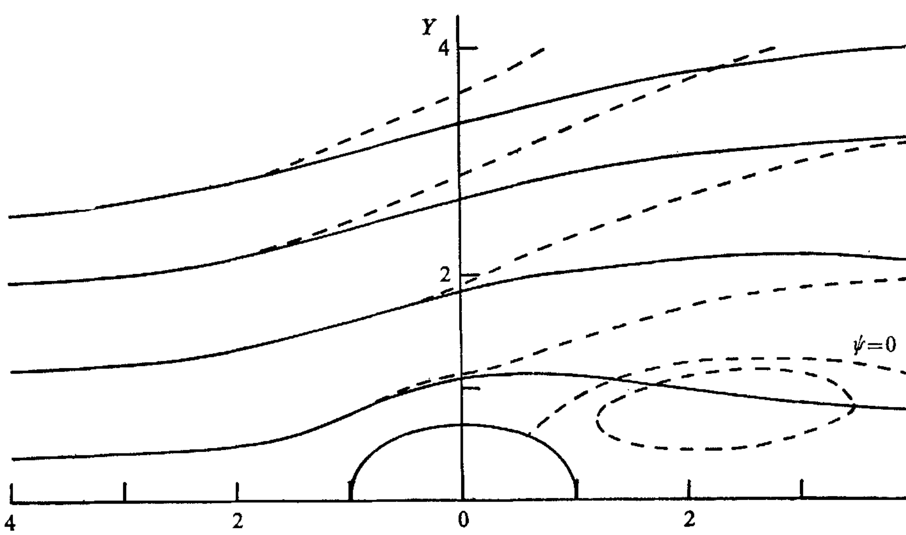

FIGURE 2. Flow lines in the upper half plane of a meridian section of an ovary ellipsoid for various values of Ψ. ———, for the case $K = 6$ and - - -, for the case $K = 10$. $e = 0{\cdot}75$. $X = x/a$, $Y = y/a$. The uniform stream and current are directed from the negative to the positive X axis.

If we now non-dimensionalize the stream functions by letting

$$\psi = \tfrac{1}{2}Ua^2\Psi, \quad \psi_0 = \tfrac{1}{2}a^2\Psi_0 \quad \text{and} \quad \psi_2 = \tfrac{1}{2}Ua^2K\Psi_2,$$

where $K = 16\pi a^3 J_0^2/(7\mu_0 U)$, (23) becomes

$$\Psi = \Psi_0 + K\Psi_2.$$

Figure 1 shows streamlines (flow lines) ψ equals ψ_0 and ψ equals ψ_2 for various values of ψ in the upper half meridian section of an ellipsoid of revolution. The eccentricity of the section is 0·75.

From figure 1 or equations (14), (22) and (23) it follows that if the streaming fluid carries a sufficiently strong current, there will be separation and flow reversal around the wake behind the ellipsoid. This was first noted by Chow (1966) for the case of a spherical obstacle. Flow lines for the ellipsoid of figure 1, representing various values of Ψ, when K is 6 and K is 10 are shown in figure 2. When K is 10 there is separation and flow reversal.

Our computations show that as e increases from zero to one, Ψ_2 becomes less significant relative to Ψ_0, and a larger value of K is required to make Ψ zero and negative, that is, as the ellipsoid becomes more elongated, we need a stronger current in order to cause separation and flow reversal.

4. Flow around a planetary ellipsoid

We again let a and c denote the polar and equatorial radii and e the eccentricity of a meridian section of the ellipsoid. The axis of the ellipsoid coincides with the x axis. We now use the transformation

$$x = \kappa\cos\theta\sinh\eta = \kappa\mu\zeta; \quad r = \kappa\sin\theta\cosh\eta = \kappa(1-\mu^2)^{\frac{1}{2}}(\zeta^2+1)^{\frac{1}{2}},$$

where $\mu = \cos\theta$ and $\zeta = \sinh\eta$. Then the ellipsoid is given by $\eta = \eta_0$ or $\zeta = \zeta_0$, where

$$a = \kappa\zeta_0, \quad c = \kappa(\zeta_0^2+1)^{\frac{1}{2}}, \quad e = (\zeta_0^2+1)^{-\frac{1}{2}}.$$

At infinity the impressed current is parallel to the axis of the ellipsoid and has intensity J_0. In terms of these quantities (see Lamb 1932)

$$\psi_1 = 2\pi J_0\kappa^2\left[1 - B_1\left(\frac{\zeta}{\zeta^2+1} - \cot^{-1}\zeta\right)\right](1-\mu^2)(\zeta^2+1), \tag{24}$$

where

$$B_1 = 1/[e(1-e^2)^{\frac{1}{2}} - \sin^{-1}e].$$

When we do an analysis similar to that of the last section we find

$$\psi_2 = [g_0(\zeta) + A + (7\mu^2-3)\,g_1(\zeta)]\,\mu(1-\mu^2), \tag{25}$$

where

$$(\zeta^2+1)\,g_0'' - 6g_0 = (7\zeta^2+3)\,G(\zeta), \tag{26}$$

$$(\zeta^2+1)\,g_1'' - 20g_1 = G(\zeta) \tag{27}$$

and

$$(\zeta^2+1)\,G'' - 6G = A_1\left[1 - B_1\left(\frac{\zeta}{\zeta^2+1} - \cot^{-1}\zeta\right)\right]. \tag{28}$$

Here

$$A_1 = 4\pi J_0^2 B_1 \kappa^5/7\mu_0.$$

The solution of (28) is

$$G = -A_1[\tfrac{1}{6} + C\{3\zeta^2+2-3\zeta(\zeta^2+1)\cot^{-1}\zeta\} \\ +\tfrac{1}{2}B_1\{\zeta-(2\zeta^2+1)\cot^{-1}\zeta+\zeta(\zeta^2+1)(\cot^{-1}\zeta)^2\}], \quad (29)$$

where C is an arbitrary constant, and the other constant of integration is set equal to zero. The solutions of (26) and (27) are given by

$$g_0 = A_1\Bigg[\frac{21\zeta^2+13}{72}+C_1\{3\zeta^2+2-3\zeta(\zeta^2+1)\cot^{-1}\zeta\} \\ +\tfrac{1}{2}C\{\zeta(\zeta^2+1)(3\zeta^2+2)\cot^{-1}\zeta-\zeta^2(3\zeta^2+4)\}-\tfrac{1}{4}B_1\zeta(\zeta^2+1) \\ -\tfrac{1}{8}B_1\{2\zeta(\zeta^4-1)(\cot^{-1}\zeta)^2-(\zeta^2+1)(4\zeta^2-3)\cot^{-1}\zeta-\zeta\}\Bigg], \quad (30)$$

and

$$g_1 = A_1[\tfrac{1}{120}+C_2\{210\zeta^4+230\zeta^2+32-30\zeta(\zeta^2+1)(7\zeta^2+3)\cot^{-1}\zeta\} \\ +\tfrac{1}{16}C\{3\zeta(\zeta^2+1)(5\zeta^2+1)\cot^{-1}\zeta-\zeta^2(15\zeta^2+13)\} \\ +\tfrac{1}{32}B_1\{\zeta(\zeta^2+1)(9\zeta^2+5)(\cot^{-1}\zeta)^2-2(\zeta^2+1)(9\zeta^2+2)\cot^{-1}\zeta+\zeta(9\zeta^2+8)\}]. \quad (31)$$

As with the ovary ellipsoid, A, C, C_1 and C_2 are determined from the vanishing of the fluid velocity on the ellipsoid, namely

$$A+g_0(\zeta_0) = g_0'(\zeta_0) = g_1(\zeta_0) = g_1'(\zeta_0) = 0.$$

The stream function ψ_0, corresponding to a streaming flow past the ellipsoid, regarded as fixed, with the general velocity U in the x direction, is (Happel & Brenner 1965)

$$\psi_0 = \tfrac{1}{2}U\kappa^2(\zeta^2+1)(1-\mu^2)\left[1-\frac{\zeta/(\zeta^2+1)-(\zeta_0^2-1)\cot^{-1}\zeta/(\zeta_0^2+1)}{\zeta_0/(\zeta_0^2+1)-(\zeta_0^2-1)\cot^{-1}\zeta_0/(\zeta_0^2+1)}\right]. \quad (32)$$

Now we set

$$\psi_0 = \tfrac{1}{2}Uc^2\Psi_0, \quad \psi_2 = \tfrac{1}{2}Uc^2K\Psi_2 \quad \text{and} \quad \psi = \tfrac{1}{2}Uc^2\Psi,$$

where

$$K = 16\pi c^3 J_0^2 B_1/(7U\mu_0)$$

and obtain

$$\Psi = \Psi_0 + K\Psi_2.$$

Figure 3 shows flow lines in the upper half plane of a meridian section of a planetary ellipsoid for the cases when K equals 2 and K equals 5.

When e is 1 we have the case of a circular disk with its plane perpendicular to the direction of the undisturbed stream at infinity. For this case

$$A = \frac{7A_1(\pi^2-12)}{18\pi^2}, \quad C = \frac{48-7\pi^2}{6\pi^2}, \quad C_1 = \frac{84-17\pi^2}{36\pi^2}, \quad C_2 = -\tfrac{1}{240}.$$

At the edge of the disk the current, like the fluid velocity in the case of inviscid flow, is infinite. The Lorentz force is zero all over the surface of the disk except at the edge, where it is indeterminate. Flow lines for the case of a disk broadside onto the undisturbed stream are shown in figure 4.

The case of streaming flow past a spherical obstacle is deduced from that of an ellipsoid, planetary or ovary, by letting e tend to zero and ζ to infinity so that $e\zeta$ equals the distance r from the centre of the sphere.

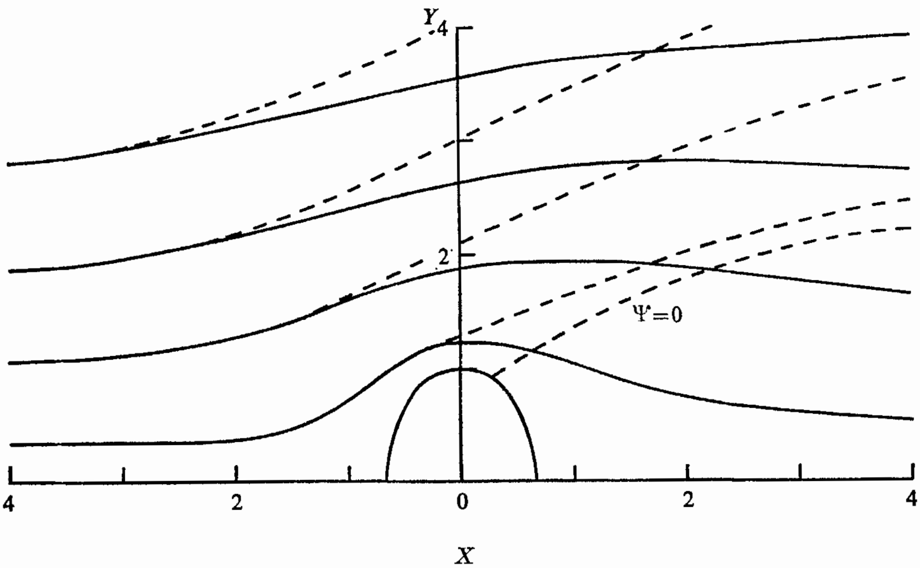

FIGURE 3. Flow lines in the upper half plane of a meridian section of a planetary ellipsoid for various values of Ψ. ———, for the case $K = 2$ and - - -, for the case $K = 5$. $e = 0{\cdot}75$, $X = x/c$, $Y = y/c$. The uniform stream and current are directed from the negative to the positive X axis.

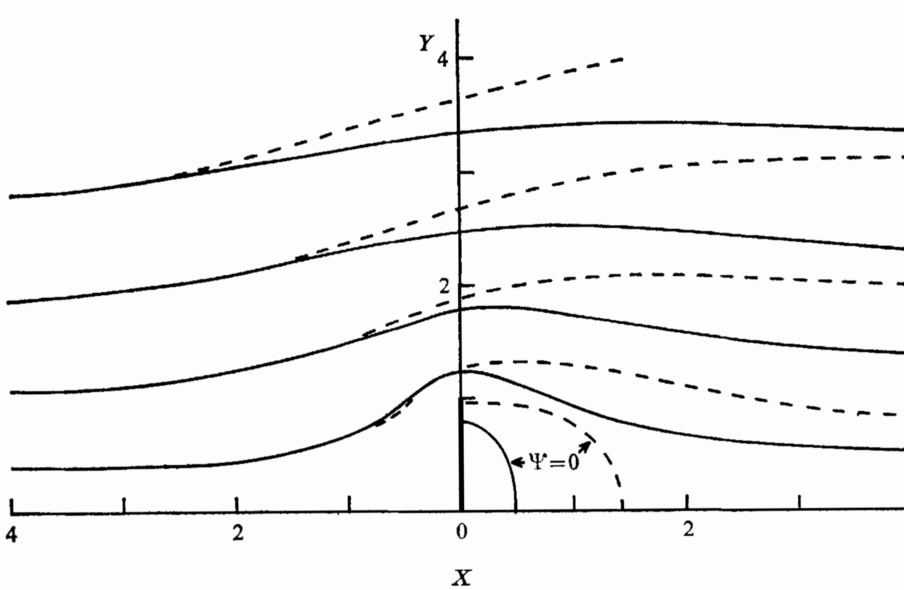

FIGURE 4. Flow lines in a half plane through the axis, which coincides with the X axis, of a disk for various values of Ψ. ———, for the case $K = 2$ and - - -, for the case $K = 5$. $X = x/c$, $Y = y/c$. The uniform stream and current are directed from the negative to the positive X axis.

Our computations show that, as the eccentricity of a planetary ellipsoid increases, the magnitude of Ψ_2 relative to that of Ψ_0 extremely close to the ellipsoid increases, and this hastens flow separation from the wake side of the ellipsoid, but everywhere else it decreases. From this and our remarks about the ovary ellipsoid when its eccentricity increases, it follows that a given current intensity will produce the maximum general flow distortion in the case of a stream past a spherical obstacle.

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